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[Annathurai, 5(6): June 2018] ISSN 2348 – 8034
DOI- 10.5281/zenodo.1291365 Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

SOME 3−REMAINDER CORDIAL GRAPHS

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ABSTRACT

Let G be a (p, q) graph. Let f be a function from V (G) to the set {1, 2, . . . , k} where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when f(u) is divided by f(v) (or) f(v) is divided by f(u) according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. Then the function f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, i, j ∈ {1, . . . , k} where $v_f(x)$ denote the number of vertices labelled with x and $|\eta_f(0) - \eta_f(1)| \leq 1$ where $\eta_f(0)$ and $\eta_f(1)$ respectively denote the number of edges labelled with an even integers and number of edges labelled with an odd integers. A graph admits a k-remainder cordial labeling is called a k- remainder cordial graph. In this paper we investigate the 3- remainder cordial labeling behavior of path, cycle, star, complete graph, comb, crown, etc,

Keywords: *Path, Cycle, Star, Complete graph, Comb, Crown.*

I. INTRODUCTION

We considered only finite and simple graphs. A comb is a caterpillar in which each vertex in the path is joined to exactly one pendant vertex. A crown Cn Θ K1 graph is obtained by joining a pendant edge to each vertex of Cn. The corona of G1 with G2, G1 ⊙ G2 is the graph obtained by taking one copy of G1 and p1 copies of G2 and joining the ith vertex of G1 with an edge to every vertex in the ith copy of G2. Cahit [1], introduced the concept of cordial labeling of graphs. Ponraj et al. [4, 6], introduced remainder cordial labeling of graphs and investigate the remainder cordial labeling behavior of path, cycle, star, bistar, complete graph, S(K1,n) , S(Bn,n) , K(1,n) ∪S(B(n,n)) , S(K(1,n)) ∪ S(B(n,n)), etc., and also the concept of k-remainder cordial labeling introduced in [5] recently. They investigate the 4-remainder cordial labeling behavior of several graphs. In this paper we investigate the 3- remainder cordial labeling behavior of path, cycle, star, complete graph, comb, crown, etc,. Terms are not defined here follows from Harary [3] and Gallian [2].

II. K- REMAINDER CORDIAL LABELING

Definition 2.1 : Let G be a (p, q) graph. Let f be a function from V (G) to the set {1, 2, . . . , k} where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when f(u) is divided by f(v) (or) f(v) is divided by f(u) according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, i, j ∈ {1, . . . , k} where $v_f(x)$ denote the number of vertices labeled with x and $|\eta_f(0) - \eta_f(1)| \leq 1$ where $\eta_f(0)$ and $\eta_f(1)$ respectively denote the number of edges labeled with an even integers and number of edges labelled with an odd integers. A graph with a k- remainder cordial labeling is called a k- remainder cordial graph.

Now we investigate the 3- remainder cordial labeling behavior of the path P_n.

[Annathurai, 5(6): June 2018] ISSN 2348 – 8034
DOI- 10.5281/zenodo.1291365 Impact Factor- 5.070

Theorem 2.2 : The path P_n is 3- remainder cordial for all n.

Proof. Let P_n be the path $u_1, u_2, \ldots, u_n$. We now give a 3- remainder cordial labeling to the path P_n. The proof of this theorem is proved in the following three cases.

Case(i): $n \equiv 0 \pmod 3$
Subcase(i): n is even.

Assign the labels 1,1, and 2 to the vertices u_1, u_2, and u_3 respectively. Next assign the labels 3,2, and 3 respectively to the vertices u_4, u_5, and u_6, and then assign the labels 1,1, and 2 to the vertices u_7, u_8, and u_9 respectively. Then next assign the labels 3,2, and 3 respectively to the vertices u_{10}, u_{11}, and u_{12}. Proceeding like this until we reach the vertex u_n. Note that in this process the last vertex u_n receive the label 3.

Subcase(ii): n is odd.

As in case(i), assign the labels to the vertices u_i, $(1 \leq i \leq n - 3)$. Finally assign the labels 1, 2, and 3 respectively to the vertices u_{n-2}, u_{n-1}, and u_n.
Thus the table 1, given below establish that this vertex labeling f is 3- remainder cordial labeling of P_n.

Table – 1

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$\eta_f(0)$	$\eta_f(1)$
n≡ 0 (mod 3) & n is even	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n-2}{2}$	$\frac{n}{2}$
n≡ 0 (mod 3) & n is odd	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$

Case(ii): $n \equiv 1 \pmod 3$

Fix the labels 3,2, and 1 to the first three vertices u_1, u_2, and u_3 and fix the labels 1,2,3, and 2 respectively to the last four vertices u_{n-2}, u_{n-2}, u_{n-1}, and u_n.

Subcase(i): n is even.

Assign the labels 1,2, and 3 to the vertices u_4, u_5, and u_6 respectively. Next assign the labels 2, 3, and 1 respectively to the vertices u_7, u_8, and u_9, and assign the labels 1,2, and 3 to the vertices u_{10}, u_{11}, and u_{12} respectively. Then assign the labels 2, 3, and 1 respectively to the vertices u_{13}, u_{14}, and u_{15}. Continuing like this until we reach the vertex u_{n-4}. Observe that in this process the last vertex u_{n-4} receive the label 3.

Subcase(ii): n is odd.

In this case, assign the labels to the vertices u_i, $(1 \leq i \leq n-4)$ in the following pattern: 1,2, 3; 2,3,1 ; ….. ; 1,2, 3; 2,3,1 respectively to the vertices u_4, u_5, u_6 ; u_7, u_8, u_9 ; …… ; $u_{n-9}, u_{n-8}, u_{n-7}$; $u_{n-6}, u_{n-5}, u_{n-4}$.
The table 2, shows that this vertex labeling f is 3- remainder cordial labeling of P_n graph for this case

Table – 2

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$\eta_f(0)$	$\eta_f(1)$
n≡ 1 (mod 3) & n is even	$\frac{n-1}{3}$	$\frac{n+2}{3}$	$\frac{n-1}{3}$	$\frac{n}{2}$	$\frac{n-2}{2}$
n≡ 1 (mod 3) & n is odd	$\frac{n-1}{3}$	$\frac{n+2}{3}$	$\frac{n-1}{3}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$

[Annathurai, 5(6): June 2018] ISSN 2348 – 8034
DOI- 10.5281/zenodo.1291365 Impact Factor- 5.070

Case(iii): $n \equiv 2 \pmod 3$

First fix the labels 3,2, and 1 to the first three vertices u_1, u_2, and u_3 and fix the labels 3, and 2 to the last two vertices u_{n-1}, and u_n respectively.

Subcase(i): n is even.

Assign the labels 1,2, and 3 to the vertices u_4, u_5, and u_6 respectively. Next assign the labels 2, 3, and 1 respectively to the vertices u_7, u_8, and u_9, and assign the labels 1,2, and 3 to the vertices u_{10}, u_{11}, and u_{12} respectively. Then assign the labels 2, 3, and 1 respectively to the vertices u_{13}, u_{14}, and u_{15}. Continuing like this until we reach the vertex u_{n-2}. Clearly in this process the vertex u_{n-2} receive the label 3.

Subcase(ii): n is odd.

Assign the labels to the vertices u_i, in the following ways: 1,2, 3; 2,3,1 ; ….. ; 1,2, 3; 2,3,1 respectively to the vertices u_4, u_5, u_6 ; u_7, u_8, u_9 ; …… ; u_{n-7}, u_{n-6}, u_{n-5}; u_{n-4}, u_{n-3}, u_{n-2}.

The table 3, establish that this vertex labeling f is 3-remainder cordial labeling of the path.

Table – 3

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$\eta_f(0)$	$\eta_f(1)$
$n \equiv 2 \pmod 3$ & n is even	$\frac{n-2}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n}{2}$	$\frac{n-2}{2}$
$n \equiv 2 \pmod 3$ & n is odd	$\frac{n-2}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$

Corollary 2.3: All cycles are 3–remainder cordial for all values of n.

Proof: The vertex labeling given in theorem 2.2, is obviously 3- remainder cordial labeling of the cycle C_n.

Next we investigate the 3- remainder cordial labeling behavior of the Star $K_{1,n}$.

Theorem 2.4 : The star $K_{1,n}$ graph is 3- remainder cordial iff $n \in \{1, 2,3,4,5,6,7,9\}$.

Proof. Let $V(K_{1,n}) = \{u, u_i : 1 \le i \le n\}$ and $E(K_{1,n}) = \{ uu_i : 1 \le i \le n \}$. Then the graph $K_{1,n}$ has n+1 vertices and n edges. Now we give a 3- remainder cordial labeling of the star.

Assign the label 2 to the n^{th} degree vertex u. The table 4 gives the 3- remainder cordial labeling of $K_{1,n}$ for $n \in \{1, 2,3,4,5,6,7,9\}$.

Table 4

u/u_i	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
1	1								
2	1	3							
3	1	3	2						
4	1	3	2	3					
5	1	3	2	3	1				
6	1	3	2	3	1	3			
7	1	3	2	3	1	3	2		
9	1	3	2	3	1	3	2	1	3

ISSN 2348 – 8034
Impact Factor- 5.070

Case(i): $n \equiv 0 \pmod 3$ and $n > 9$.

Let $n = 3t$.

Subcase(i): $f(u) = 1$.

In this case all the edges received the label 0. That is $\eta_f(0) = n$. which is a contradiction.

Subcase(ii): $f(u) = 2$.

Clearly $\eta_f(0) \geq t-1 + t = 2t-1$, which is a contradiction.

Subcase(iii): $f(u) = 3$.

Similar to subcase(ii).

Case(ii): $n \equiv 1 \pmod 3$ and $n > 7$.

Let $n = 3t + 1$.

Subcase(i): $f(u) = 1$.

In this case all the edges received the label 0. That is $\eta_f(0) = n$. which is again a contradiction.

Subcase(ii): $f(u) = 2$ (or) 3.

In this case

$\eta_f(0) \geq 2t$, which is a contradiction to size of $K_{(1,n)}$ is $3t+1$.

Case(iii): $n \equiv 2 \pmod 3$ and $n > 6$.

As in case(i), we get a contradiction.

Theorem 2.5: The complete graph K_n is 3- remainder cordial iff $n \leq 3$.

Proof: The graphs K_1, K_2 are 3- remainder cordial follows from the theorem 2.2 and K_3 is 3- remainder cordial by corollary 2.3.

Case(i): $n \equiv 0 \pmod 3$

Let $n = 3t$ where $t > 1$. Suppose the function f is 3- remainder cordial labeling of K_n. This implies that $v_f(1) = v_f(2) = v_f(3) = t$. Then clearly $\eta_f(1) = t^2$ and $\eta_f(0) = \binom{t}{2} + \binom{t}{2} + \binom{t}{2} + t^2 + t^2$

$= 3\binom{t}{2} + 2t^2$

$= 3\frac{t(t-1)}{2} + 2t^2$

We get $\eta_f(0) - \eta_f(1) = 3\frac{t(t-1)}{2} + 2t^2 - t^2 = 3\frac{t(t-1)}{2} + t^2$.

Therefore $|\eta_f(0) - \eta_f(1)| > 1$, a contradiction to the definition of k- remainder cordial labeling.

Case(ii): $n \equiv 1 \pmod 3$

Let $n = 3t + 1$ where $t \geq 1$. We have the following types.

Type A: $v_f(1) = t+1$, $v_f(2) = t$, $v_f(3) = t$

Type B: $v_f(1) = t$, $v_f(2) = t+1$, $v_f(3) = t$

Type C: $v_f(1) = t$, $v_f(2) = t$, $v_f(3) = t+1$

Subcase(i): Type A: $v_f(1) = t+1$, $v_f(2) = t$, $v_f(3) = t$

We find $\eta_f(1) = t^2$ and $\eta_f(0) = \binom{t+1}{2} + \binom{t}{2} + \binom{t}{2} + t(t+1) + t(t+1)$

$= \frac{t(t+1)}{2} + 2\frac{t(t-1)}{2} + 2t^2 + 2t$

$= \frac{3t^2+3t}{2}$

Then we have $\eta_f(0) - \eta_f(1) = \frac{3t^2+3t}{2} - t^2 = \frac{t^2+3t}{2}$.

Therefore $|\eta_f(0) - \eta_f(1)| > 1$, a contradiction to the definition of k- remainder cordial labeling.

Subcase(ii): Type B: $v_f(1) = t$, $v_f(2) = t+1$, $v_f(3) = t$

We find $\eta_f(1) = t(t+1)$ and $\eta_f(0) = \binom{t}{2} + \binom{t+1}{2} + \binom{t}{2} + t(t+1) + t^2$

$= 2\frac{t(t-1)}{2} + \frac{t(t+1)}{2} + 2t^2 + t$

$= \frac{7t^2-3t}{2}$

Then we get $\eta_f(0) - \eta_f(1) = \frac{7t^2-3t}{2} - t^2 - t = \frac{5t^2-5t}{2}$.

Finally we have $|\eta_f(0) - \eta_f(1)| = \frac{5t^2-5t}{2} > 1$, a contradiction to edge condition of k- remainder cordial labeling.

Subcase(iii): Type C: $v_f(1) = t$, $v_f(2) = t$, $v_f(3) = t+1$

We find $\eta_f(1) = t(t+1)$ and $\eta_f(0) = \binom{t}{2} + \binom{t}{2} + \binom{t+1}{2} + t^2 + t(t+1)$

$= 2\frac{t(t-1)}{2} + \frac{t(t+1)}{2} + 2t^2 + t$

$= \frac{2t^2-2t+t^2+t+4t^2+2t}{2}$

$= \frac{7t^2+t}{2}$

Then we get $\eta_f(0) - \eta_f(1) = \frac{7t^2+t}{2} - t^2 - t = \frac{5t^2-t}{2}$.

Clearly we get $|\eta_f(0) - \eta_f(1)| = \frac{5t^2-t}{2} > 1$, a contradiction to the definition of k- remainder cordial labeling.

Case(iii): $n \equiv 2 \pmod 3$

Let $n = 3t + 2$ where $t \geq 1$. We have the following three types.

Type D: $v_f(1) = t$, $v_f(2) = t+1= v_f(3)$

Type E: $v_f(1) = t+1= v_f(2)$, $v_f(3) = t$

Type F: $v_f(1) = t+1= v_f(3)$, $v_f(2) = t$

Subcase(i): Type D: $v_f(1) = t$, $v_f(2) = t+1= v_f(3)$

We find $\eta_f(0) = \binom{t}{2} + \binom{t+1}{2} + \binom{t+1}{2} + t(t+1) + t(t+1)$

$= \frac{t(t-1)}{2} + 2\frac{t(t+1)}{2} + 2t^2 + 2t$

$= \frac{7t^2-5t}{2}$

and $\eta_f(1) = (t+1)^2 = t^2 + 2t+1$.

Then we get $\eta_f(0) - \eta_f(1) = \frac{7t^2-5t}{2} - (t^2 + 2t+1) = \frac{5t^2-t-2}{2}$.

This implies $|\eta_f(0) - \eta_f(1)| = \frac{5t^2-t-2}{2} > 1$, a contradiction to $|\eta_f(0) - \eta_f(1)| \leq 1$.

Subcase(ii): Type E: $v_f(1) = t+1= v_f(2)$, $v_f(3) = t$

We get $\eta_f(0) = \binom{t+1}{2} + \binom{t+1}{2} + \binom{t}{2} + (t+1)^2 + t(t+1)$

$= 2\frac{t(t+1)}{2} + \frac{t(t-1)}{2} + t^2 + 2t+1+ t^2 + t$

$= \frac{7t^2+7t+2}{2}$

and $\eta_f(1) = t(t+1) = t^2 + t$.

Then we get $\eta_f(0) - \eta_f(1) = \frac{7t^2+7t+2}{2} - (t^2 + t) = \frac{5t^2-5t+2}{2}$.

Clearly $|\eta_f(0) - \eta_f(1)| = \frac{5t^2-5t+2}{2} > 1$, a contradiction of k- remainder cordial labeling definition.

Subcase(iii): Type F: $v_f(1) = t+1= v_f(3)$, $v_f(2) = t$

We have $\eta_f(0) = \binom{t+1}{2} + \binom{t}{2} ++ \binom{t+1}{2} + (t+1)^2 + t(t+1)$

$= 2\frac{t(t+1)}{2} + \frac{t(t-1)}{2} + t^2 + 2t+1+ t^2 + t$

$= \frac{7t^2+7t+2}{2}$

and $\eta_f(1) = t(t+1) = t^2 + t$.

Then we get $\eta_f(0) - \eta_f(1) = \frac{7t^2+7t+2}{2} - (t^2 + t) = \frac{5t^2-5t+2}{2}$.

Therefore $|\eta_f(0) - \eta_f(1)| = \frac{5t^2-5t+2}{2} > 1$, a contradiction to the definition of k- remainder cordial labeling. Thus , K_n is 3- remainder cordial iff $n \le 3$.

Finally we investigate the 3- remainder cordial labeling behavior of the comb.

Theorem 2.6: The comb $P_n \odot K_1$ is 3- remainder cordial for all values of n.

Proof. Let P_n be a path $u_1, u_2, \ldots, u_n$. Let $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \le i \le n\}$ and $E(P_n \odot K_1) = \{ u\, v_i : 1 \le i \le n\}$. It easy to verify that the graph $P_n \odot K_1$ has 2n vertices and 2n – 1 edges respectively.

Case(i): $n \equiv 0 \pmod 3$

Assign the labels 1,2, and 3 to the vertices u_1, u_2, and u_3 respectively. Next assign the labels 1,2, and 3 respectively to the vertices u_4, u_5, and u_6. Continuing like this until we reach the vertex u_n. Clearly in this process the last vertex u_n receive the label 3. Next we move to the pendant vertices v_i , $(1 \le i \le n)$. Assign the labels 1,3, and 2 to the vertices v_1, v_2, and v_3 respectively. Next assign the labels 1,3, and 2 respectively to the vertices v_4, v_5, and v_6. Proceeding like this until we reach the vertex v_n. So that in this process the last vertex v_n receive the label 2.

Case(ii): $n \equiv 1 \pmod 3$

Assign the labels to the vertices u_i, v_i , $(1 \le i \le n-1)$ as in case(i). Next assign the labels 2, and 1 respectively to the vertices u_n, and v_n.

Case(iii): $n \equiv 2 \pmod 3$

Assign the labels to the vertices u_i, v_i , $(1 \le i \le n-2)$ as in case(i). Next assign the labels 2, and 1 respectively to the vertices u_n, and v_n. Finally assign the labels 2, 1, 3, and 1 to the vertices u_{n-1}, u_n, v_{n-1} and v_n respectively. The table 5 shows that the function f is 3–remainder cordial labeling of the comb.

Table 5

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$\eta_f(0)$	$\eta_f(1)$
$n \equiv 0 \pmod 3$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$	n-1	n
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	n-1	n
$n \equiv 2 \pmod 3$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	n-1	n

Corollary 2.7: All crowns are 3- remainder cordial for all values of n.

Proof: Let $C_n \odot K_1$ be the given crown and C_n: $u_1\, u_2 \ldots, u_n\, u_1$ be the cycle. The vertex labeling given in theorem: 2.6 is obviously a 3- remainder cordial labeling of $C_n \odot K_1$.

For illustration, a 3- remainder cordial labeling of $C_6 \odot K_1$ is shown in Figure 2.1.

ISSN 2348 – 8034
Impact Factor- 5.070

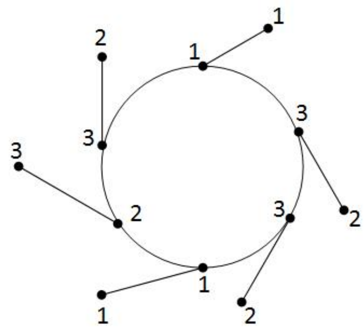

Figure 2.1

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